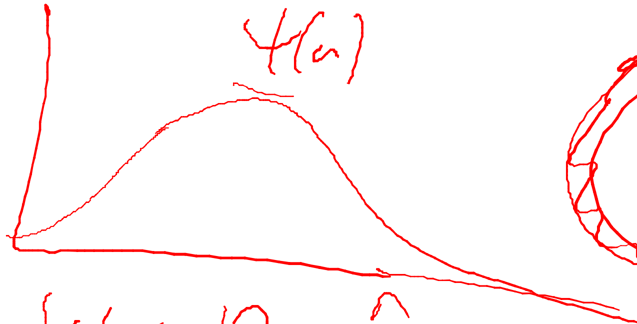
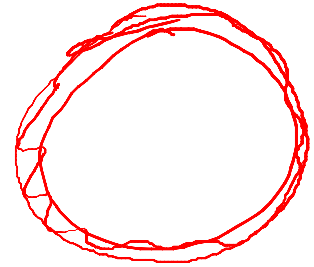
Physics 214 Final Exam Review Problems

The following questions are designed to give you some practice with concepts covered <u>since the midterm</u>. You should look at old practice midterms for sample problems covering the earlier course material. Some are specifically designed to be difficult in order to make sure you can go beyond simple “plug and chug” problems.

1. An electron has a wavefunction $\Psi(r, \theta, \phi) = Cr^3e^{-r/a}$. At what radius is one most likely to find the electron?

a. r = a
b. r = 2a
c. r = 3a
d. r = 4a
e. r = 5a

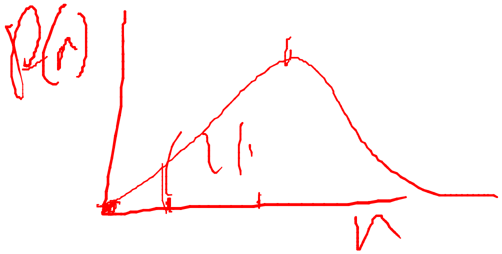




$$P(a<r<b) = \int_a^b dVol\,|\psi(r)|^2$$

$$dVol = 4\pi r^2 dr$$

$$\approx 4\pi \int r^2 r^6 e^{-2r/a}\,dr \qquad P(r)$$



$$\frac{d}{dr}\left(r^8 e^{-2r/a}\right) = 0$$

$$= 8r^7 e^{-2r/a} + r^8 e^{-2r/a}\left(-\frac{2}{a}\right) = 0$$

$$= 8 - \frac{2r}{a} = 0$$

$$\frac{2r}{a} = 8 \qquad r = \frac{8a}{2} = 4a$$

1'. What happens to this radius if one increases the charge of the nucleus?

a. decrease
b. increase
c. stay the same

$$a_0 = \frac{4\pi\epsilon_0 \hbar^2}{m_e e^2}$$

$\Rightarrow e_{electron} (e_{nucleus})$

1''. What happens to this radius if replace the electron by a muon (forming 'muoniom')? A muon is essentially a heavy electron: $m_{muon} \sim 200\ m_e$

a. decrease
b. increase
c. stay the same

2. This electron is in what orbital angular momentum state?

a. s
b. p
c. d

$$\boxed{\Psi(r,\theta,\phi) = Cr^3e^{-r/a}} \neq \Psi(\theta,\phi)$$

Spherically symmetric

$\ell = 0$ (s = 0)

$$\Psi(r,\theta,\phi) = R_{n\ell}(r)\, Y_{\ell m}(\theta,\phi)$$

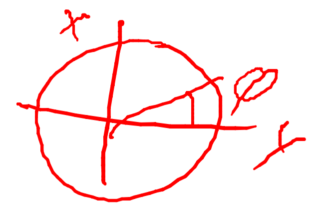
$$Y_{00} = \frac{1}{\sqrt{4\pi}}$$

$$Y_{\ell=1,m=0} = \sqrt{\frac{3}{4\pi}}\cos\theta \quad \cancel{e^{i(m=0)\phi}}$$

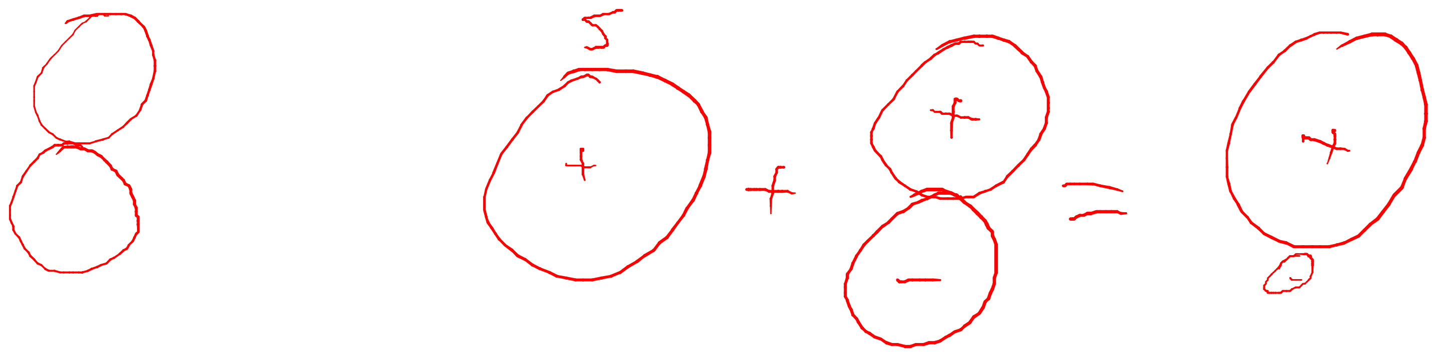
$$Y_{\ell=1,m=\pm1} = \mp\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{\pm i\phi}$$

$$e^{i\phi}e^{-i\omega t} = e^{i(\phi-\omega t)} \Rightarrow \text{circulating}$$

$L_z = m\hbar$



2’. The previous problem had a spherically symmetric wavefunction. We can also have wavefunctions that have various lobes. However, even in these cases, the electron is still equally likely to found in the top half plane, or the bottom half plane (or in any two hemispheres). How can we get an electron that is more likely to be, e.g., above the nucleus?



Does it move?

$$e^{-i\omega_1 t}\psi_{100} + \psi_{210}\, e^{-i\omega_2 t}$$

$$e^{-i\omega_2 t}\psi_{200} + \psi_{210}\, e^{-i\omega_2 t}$$

$\frac{-13.6\,eV\,Z^2}{n^2} = E$ in hydrogenic atom only depends on n

3. An electron in a hydrogen atom is in a p state. Which of the following statements is true?

$\ell = 1$

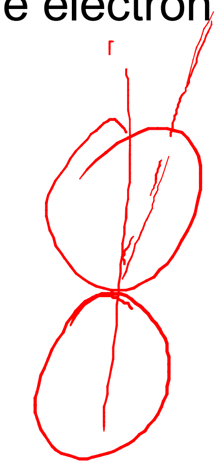
a. The electron has a total angular momentum of $\hbar$. $L^2 = \ell(\ell+1)\hbar^2 = 2\hbar^2$

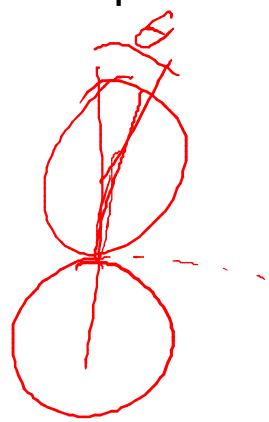
b. The electron has an energy of -13.6 eV. $n = 1 \quad \ell < n$

c. The probability to find the electron within 0.1 nm of the origin changes in time.

d. The electron's wave function has at least one node (i.e., at least one place in space where it goes to zero).

e. The electron has a z-component of angular momentum equal to $\sqrt{2}\hbar$.





$P = \int 4\pi r^2 dr \left| \psi(r,\theta,\phi) e^{-i\omega t} \right|^2$

$\ell = 1 \quad m_\ell = -1, 0, +1$

$L_z = m_\ell \hbar = 0, \pm\hbar$

Problems 4,5, and 6 are related.

4. An electron in an infinite square well of width L = 1 nm has the wavefunction:

$$\psi(x) \propto \sqrt{\frac{2}{L}}\left[\sin\left(\frac{3\pi x}{L}\right) + \sin\left(\frac{5\pi x}{L}\right) - 2\sin\left(\frac{\pi x}{L}\right)\right]$$

$n=3$, $n=5$, $n=1$

$\sin\left(\frac{n\pi x}{L}\right)$

What is/are the possible result/results for a measurement of the electron's energy?

a. 0.376 eV
b. 2.38 eV
c. 0.376 eV, 3.39 eV, or 9.41 eV
d. 11.3 eV
e. 12.1 eV

$E_n = \frac{h^2}{8mL^2} n^2 = \left(\frac{h^2}{2m}\right)\frac{1}{4L^2} n^2$

$\frac{1.505\,\text{eV}\cdot\text{nm}^2}{4\,\text{nm}^2} n^2$

$E_1 = \frac{1.505}{4} = 0.376\,\text{eV}$

$E_3 = E_1 9 = 3.39\,\text{eV}$

$E_5 = E_1 25 = 9.41\,\text{eV}$

$\left(E_2 = 4E_1 = 1.504\,\text{eV}\right)$

5. What is the probability of measuring the electron in the previous problem to have an energy of 0.376 eV?

a. 4
b. 0.67
c. -0.67
d. 0.5
e. 0

$$\psi(x) \propto \sqrt{\frac{2}{L}}\left[\underset{\tilde{C}}{\sin(\frac{3\pi x}{L})} + \underset{\tilde{B}}{\sin(\frac{5\pi x}{L})} - \underset{\tilde{A}}{2\sin(\frac{\pi x}{L})}\right]$$

$$P \propto \tilde{A}^2 \qquad \tilde{A} = -2$$

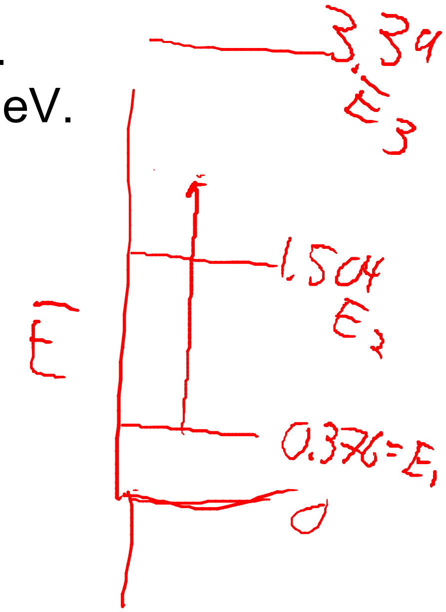
$$|A|^2 + |B|^2 + |C|^2 = 1$$

$$P(1) = \frac{|\tilde{A}|^2}{|\tilde{A}|^2 + |\tilde{B}|^2 + |\tilde{C}|^2} = \frac{(-2)^2}{(-2)^2 + 1^2 + 1^2} = \frac{4}{4+1+1} = \frac{2}{3}$$

$$P(3) = \frac{1^2}{6} = \frac{1}{6} = P(5)$$

6. If indeed we measure the electron to have energy 0.376 eV, and then we shine on light of wavelength 824.5 nm, what will happen?

a. The electron will be excited to a state with energy 1.75 eV.
b. The electron will be excited to the state with energy 1.504 eV.
c. The electron will not be excited.



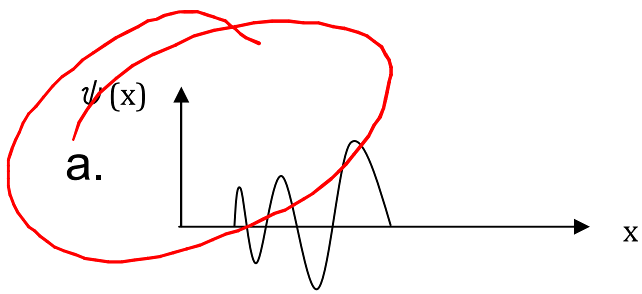
$$E_{photon} = hf = \frac{hc}{\lambda} = \frac{1240\,eV\text{-}nm}{824.5\,eV}$$

$$= 1.504\,eV$$

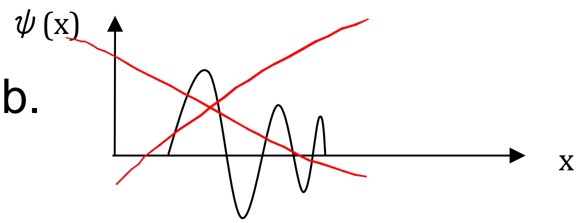
$$E_{photon} + E_1 = E_2$$

7. A particle is trapped in the potential well below.
Which of the wave functions most closely describes the particle?

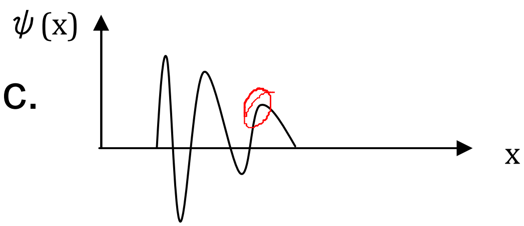
a.

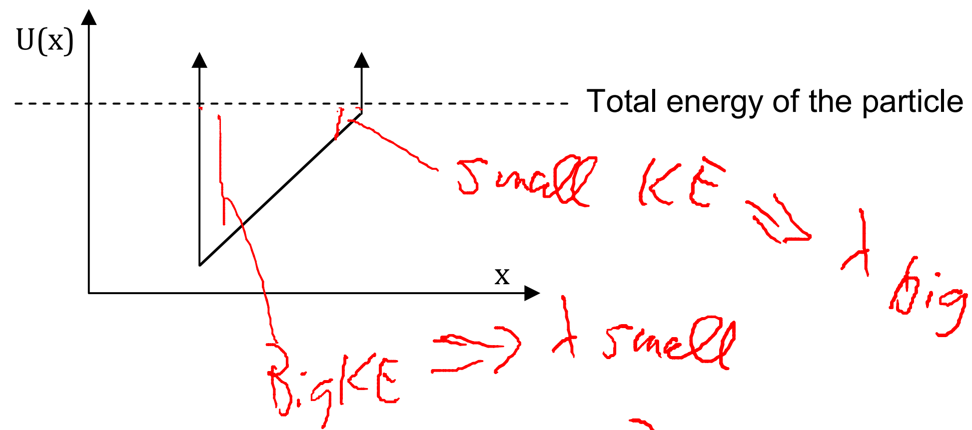


b.

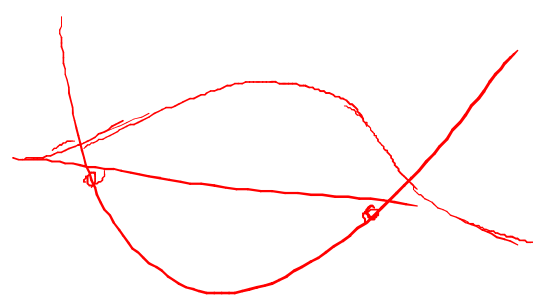


c.





$$KE = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}$$



8. What state is this particle in (where n = 1 is the ground state)?

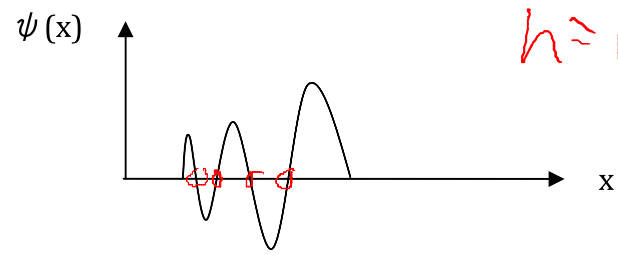
a. n = 2

b. n = 3

c. n = 4

d. n = 5

e. n = 6



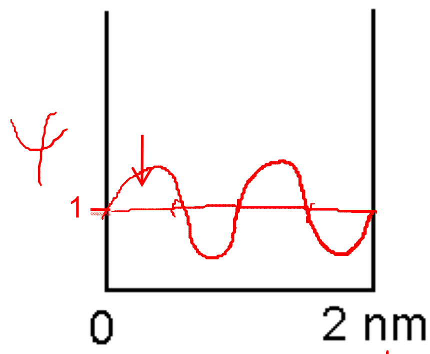
$n=1 \Rightarrow$ no zero-crossing

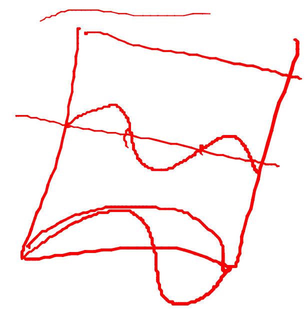
9. An electron is in the 3rd excited state of a 2-nm wide infinite square well. What is the probability of measuring the electron to be between x = 0.23 nm and x = 0.27 nm?

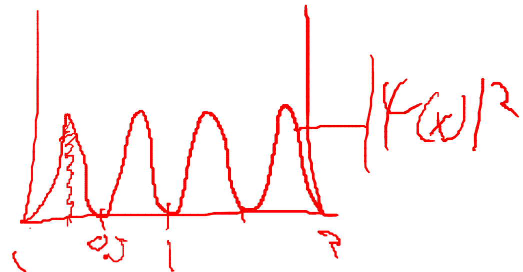
a. 0.04
b. 0.10
c. 0.16
d. 0.32
e. 0.64

n = 4

$\sqrt{\frac{2}{L}} \sin \frac{4\pi}{L} x$

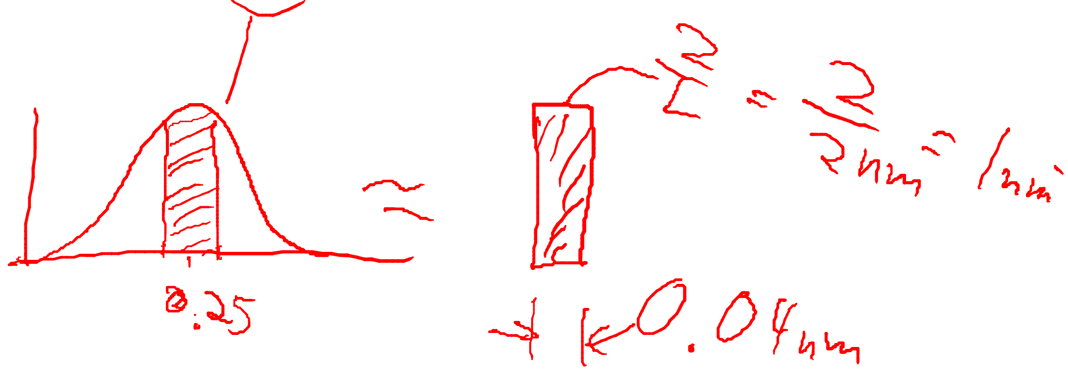






$\int_{0.23}^{0.27} |\psi|^2 dx$

Finite



10. An electron with total energy E approaches a barrier of height U_0 and width L. Assuming $E<U_0$, which one of the following changes will **increase** the probability for the electron to appear on the other side of the barrier?

~~a.~~ increase L
b. increase E
~~c.~~ increase U_0

$$T = G\, e^{-2KL}$$

$$G = 16\frac{E}{U_0}\left(1-\frac{E}{U_0}\right)$$

$$K = \sqrt{\frac{(U_0-E)\,2m}{\hbar^2}}$$

11. Which of the following normalized wave functions for the infinite square well has the shortest period of oscillation in time?

a. (sin(π x/L) + sin(2 π x/L)) / sqrt(L) $n=1$ $n=2$
b. (sin(2 π x/L) + sin(3 π x/L)) / sqrt(L)
c. (sin(π x/L) + sin(3 π x/L)) / sqrt(L) $n=3$ 10

$T = \frac{1}{f} = \frac{h}{E_n - E_m}$

$f = \Delta f = f_n - f_m = \frac{E_n - E_m}{h}$

Smallest T

$\Rightarrow$ Largest $\Delta f \Rightarrow$ largest $\Delta E = E_n - E_m$

$E = \frac{h^2}{8mL^2} n^2$

$\Delta E = \frac{h^2}{8mL^2}(n^2 - m^2)$

$n=5$
$n=3$
$25 - 9 = 16$

a $2^2 - 1^2 = 3$
b. $3^2 - 2^2 = 5$
c. $3^2 - 1^2 = 8$

Independent of amplitudes

11'. Let's say the electron is in the state (sin(π x/L) + sin(2 π x/L)) / sqrt(L)? If we measure the energy, what will we get?

n=1 n=2 $\sqrt{\frac{2}{L}}\left(\frac{\sin + \sin}{\sqrt{2}}\right)$

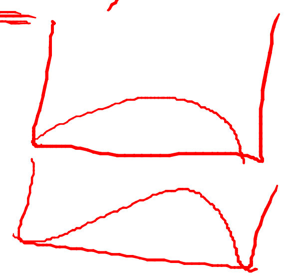
a. $h^2/8mL^2$
b. $4h^2/8mL^2$
c. $(5/2)h^2/8mL^2$ ⇒ average energy

$\frac{h^2}{8mL^2}(h^2)$

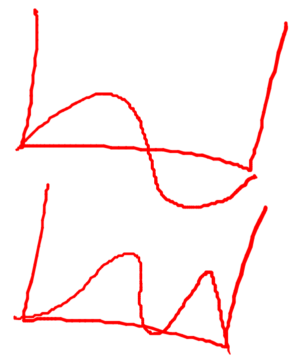
11''. What if we now measure which *side* of the well the electron is?

a. P(left) > P(right)
b. P(left) < P(right)
c. P(left) = P(right)

a) E_1 ⇒



b) E_2



11'''. Let's say we measured the particle on the left. What now might we see if we measure the energy again?

a. $h^2/8mL^2$
b. $4h^2/8mL^2$
c. $(5/2)h^2/8mL^2$

Δx small ⇒ big Δp

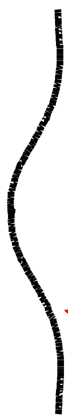
big ΔE

Can only measure allowed E_j: $\frac{h^2}{8mL^2}$ h^2

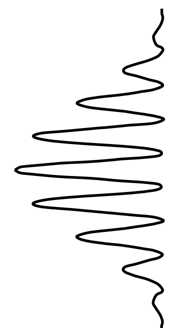
Problems 12 and 13 are related.

12. Which of the following probability distributions will you observe from a beam of electrons passing through a double slit with one slit covered? (Assume that the detection screen is far away from the slits, i.e, the diagrams are not drawn to scale)?

a.



b.



Wider distribution if λ bigger

$p = \frac{h}{\lambda}$

$\Rightarrow p$ smaller $= mv$

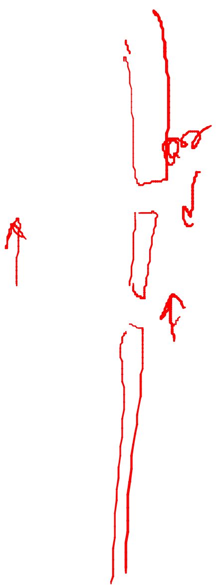
v smaller

Diffraction pattern from 1 slit

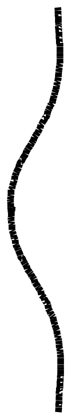
Narrow slit $\Rightarrow$ wide distribution

$\Delta x \Delta p_x \geq h$

13. Now both slits are unblocked. However, we modify the experiment in the following way: We prepare the electrons incident on the slits so that they <u>all</u> have their spins “pointing up”, i.e., so that $m_s = +1/2$. We install a tiny radio-coil near the top slit (this is only a thought experiment!), so that the spin of any electron that passes through the top slit is flipped (without affecting the spin of electron passing through the bottom slit). Now which pattern do we see?

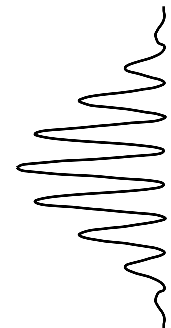


a.



Distinguishable processes don't interfere

b.



14. What frequency of electromagnetic radiation will flip a “spin up” electron to a “spin down” electron in a magnetic field of 2.0 T?

a. 2.4×10^{9} Hz
b. 4.1×10^{9} Hz
c. 5.6×10^{10} Hz
d. 7.1×10^{11} Hz
e. 8.8×10^{12} Hz

Charged spin ⇒ μ

$U = -\vec{\mu} \cdot B$

$\mu = \frac{e\hbar}{2m}$

B_z

$\downarrow$ — $+\mu B$

$\uparrow$ — $-\mu B$

s

$\vec{\mu}$

$E_{photon} = \Delta E = \mu B - -\mu B = 2\mu B = hf$

$$f = \frac{2\mu B}{h} = \frac{2(9.28 \times 10^{-24}\ J/T)(2T)}{6.6 \times 10^{-34}\ J \cdot s}$$

15. A photon has energy 3 eV. What is its momentum?

a. 0
b. $1.6*10^{-27}$ kg m/s
c. $9.4*10^{-34}$ kg m/s

$$E = \frac{hc}{\lambda} = \frac{1240\,\text{eV-nm}}{\lambda}$$

$$p = \frac{h}{\lambda}$$

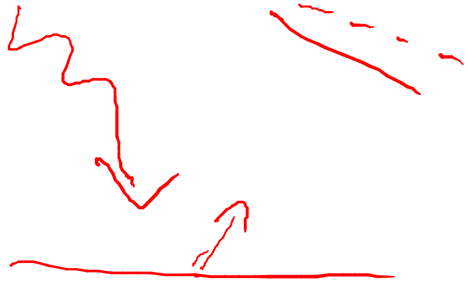
$$\lambda = \frac{1240}{3} = 413\,\text{nm}$$

$$\left(p = \frac{h}{\lambda} = \frac{hf}{c} = \frac{E}{c} \qquad \lambda f = c \qquad E = pc \right)$$

16. A laser with wavelength 300 nm illuminates a metal in a photoelectric effect experiment. It takes a stopping potential of 2 Volts to halt the ejected electrons. What is the work function of the metal?

a. 1.0 eV
b. 2.1 eV
c. 3.2 eV

$E = \frac{h^2}{2m\lambda^2}$



$E_{photon} = \phi + KE$

$= \phi + eV_{stop}$

$\phi = E_{photon} - eV_{stop}$

$= \frac{1240 \text{ eV-nm}}{300} - 2\text{ eV}$

$= 2.1 \text{ eV}$

Largest λ to eject?

Smallest $E_{photon} = \phi = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{\phi} = \frac{1240}{2.1} = 590 \text{ nm}$

How does λ_{dB} of emitted electron depend on E_{photon}, ϕ, V_{stop}?

$\lambda \sim \frac{1}{\sqrt{KE}} = \frac{1}{\sqrt{E_{phot} - \phi}} = \frac{1}{\sqrt{eV_{stop}}}$

16'. Assume a laser with wavelength 300 nm illuminates a metal with a work function 2.1 eV. Assuming every photon liberates one electron, how many electrons are released each second if the laser has a power of 1 mW?

a. 1.5×10^{15}
b. 2.5×10^{16}
c. 3.5×10^{17}

$$\#elect/sec = \#phot/s = \frac{Power}{E_{photon}} = \frac{10^{-3}\,J}{4.1\,eV} \times \frac{1\,eV}{1.6\times10^{-19}\,J}$$

$$Power = E_{photon}\,\frac{\#phot}{s}$$

$$\#phots = \frac{Power}{hc/\lambda}$$

$$\frac{1240\,eV\,nm}{300\,nm} = 4.1\,eV$$

16''. What if we keep the power fixed, but use a laser with half the wavelength (i.e., 150 nm)?

a. $N_{emitted}$ stays the same
b. $N_{emitted}$ decreases
c. $N_{emitted}$ increases

#phot/s decreases because E_{photon} increases

16'''. What if we keep the power fixed, but use a laser with twice the wavelength (i.e., 600 nm)?

a. $N_{emitted}$ stays the same
~~b. $N_{emitted}$ decreases~~
c. $N_{emitted}$ increases

$$\frac{4.1\,eV}{2} < 2.1\,eV$$

No electrons emitted!

Problems 17 and 18 are related.

17. An electron is confined to a rectangular region in space with sides L_x =2 nm, L_y = 3 nm, L_z = 2 nm. What is the energy of the ground state?

a. 0.094 eV

b. 0.19 eV

c. 0.23 eV

$$E = \frac{h^2}{2m}\frac{1}{4}\left[\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} + \frac{n_z^2}{L_z^2}\right]$$

$(1, 1, 1)$

$$\frac{1.505\ \text{eV nm}^2}{4}\left[\frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{2^2}\right] \checkmark$$

$(1\ 2\ 1) = \frac{1}{4} + \frac{4}{9} + \frac{1}{4}$

$(2\ 1\ 1) = \frac{4}{4} + \frac{1}{9} + \frac{1}{4}$

18. What is the degeneracy of the 1[st] excited state for the electron in the previous problem (neglecting the effect of spin)?

a. 1
b. 2
c. 3

$L_x = 2$
$L_y = 3$
$L_z = 2$

	l	with spin
(2 1 1) (1 1 2)	2	4
(1 2 1)	1	2
(1 1 1)	1	2

18’. How many electrons can the well hold, and still not have any in the third excited state (now including spin effects)?

a. no limit
b. 4
c. 8
d. 9
e. 10

2nd excited (2 1 1) (1 1 2) 4

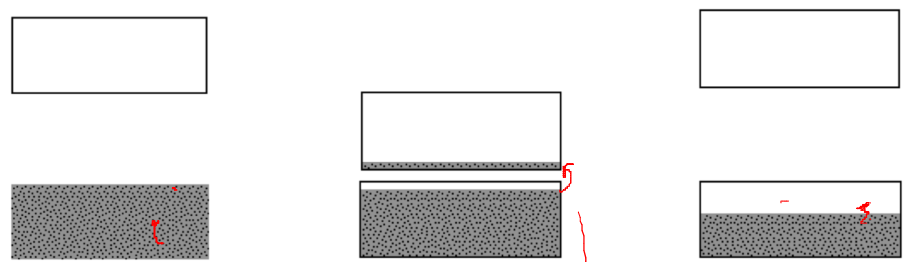
1st (1 2 1) 2

9 (1 1 1) 2

8 electrons

No spin

19. Which of the following energy band pictures corresponds to a conductor?



a. b. c.

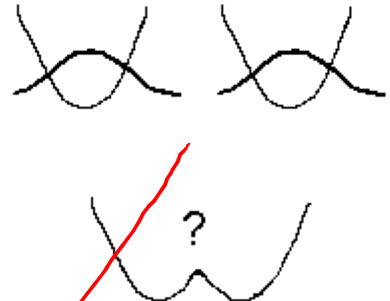
Insulator

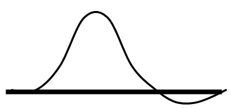
Semiconductor (T > 0)

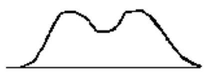
Metal / Conductor

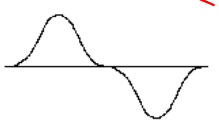
Problems 20 and 21 are related.

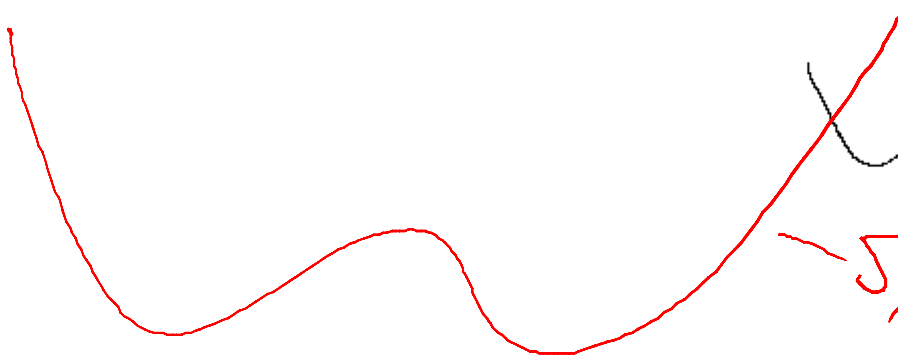
20. Two harmonic oscillators in their ground states are brought near each other. Which of the following pictures shows the correct 1st excited state for the combined system?



a. 

b. 

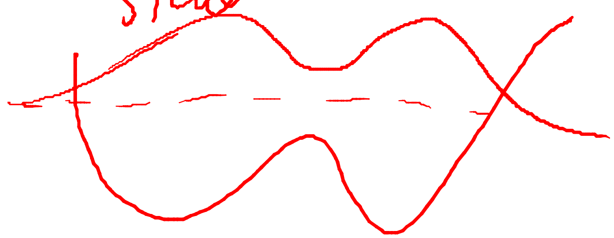
c. 



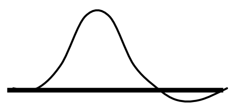
Symmetric Potential

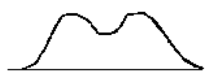
Even & Odd Solution

Ground state

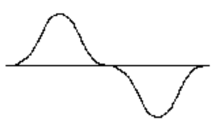


21. Assume there is one electron from each harmonic oscillator (and neglect electrostatic interactions between the electrons). If the “molecule” is in its lowest energy state, one of the electrons is in state (b.) above. Which of the above pictures is appropriate for the wave function of the second electron?

a. 

b.  2nd electron with opposite spin

"bonding orbital"

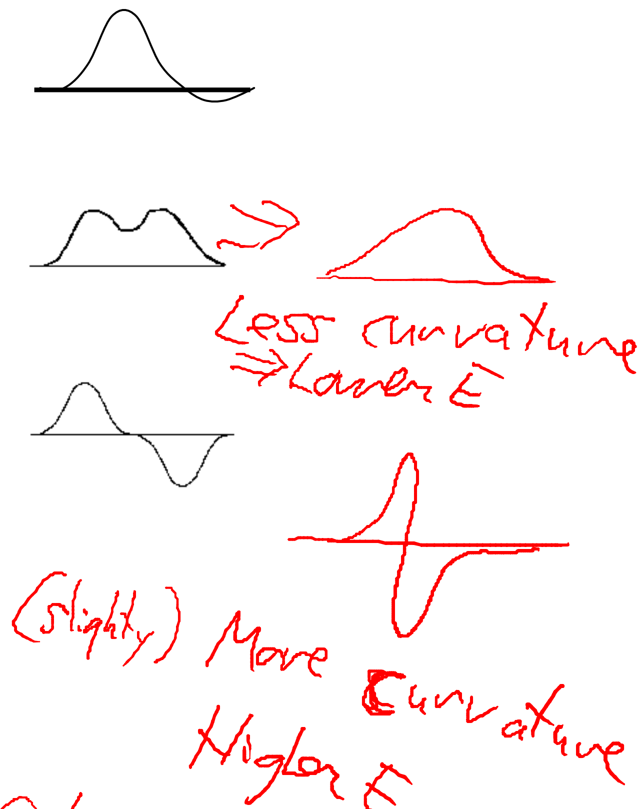
c. 

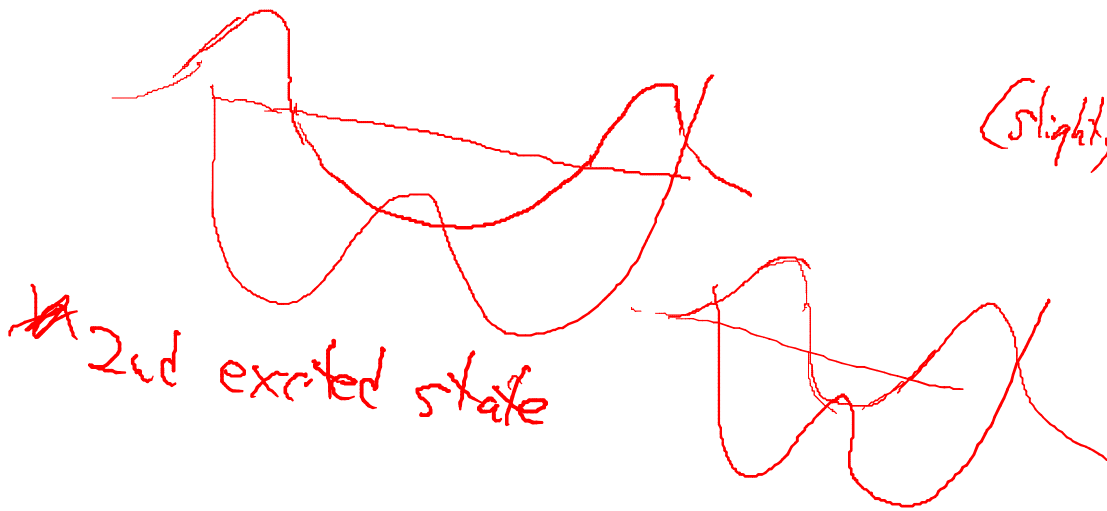
21’. If we allow the two wells to move closer together, how does the energy of the ground state change?

a. decreases

b. increases

c. stays the same





22. A beam of electrons is sent toward a potential barrier (height = 2 eV) with velocity 6×10^5 m/s. If 97.5% of the incident beam is reflected, what is the width of the barrier?

a. 0.01 nm
b. 0.05 nm
c. 0.1 nm
d. 0.5 nm
e. 1 nm

$$T = 1 - R = 1 - 0.975 = 0.025 = G e^{-2KL}$$

$$L = \frac{-1}{2K} \ln\left(\frac{T}{G}\right)$$

$$G = 16 \frac{E}{U_0}\left(1 - \frac{E}{U_0}\right) = 16 \cdot \frac{1}{2}\left(1 - \frac{1}{2}\right) = 4$$

$$K = 2\pi \sqrt{\frac{(U-E)}{h^2/2m}}$$

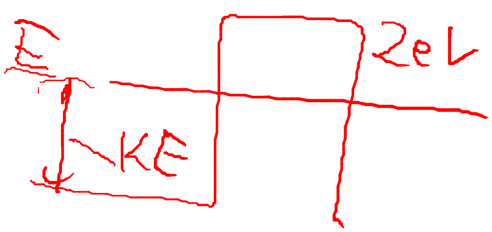
$$= 2\pi \sqrt{\frac{(2-1)\,eV}{1.505\, eV\text{-}nm^2}}$$

$$= 5.12\, nm^{-1}$$

$$E = \frac{1}{2} m v^2$$

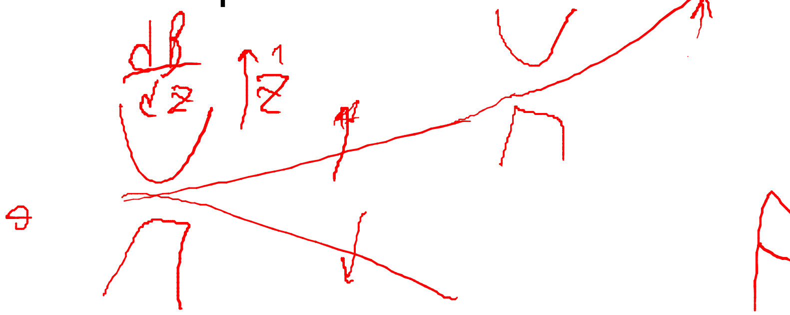
$$= \frac{1}{2}(9.1 \times 10^{-31}\, kg)(6 \times 10^5)^2$$

$$= 1.6 \times 10^{-19}\, J = 1\, eV$$



Problems 23 and 24 are related. $\ell = 0$

23. A hydrogen atom in its ground state traveling in the +x-direction is passed ~~along the~~ through a Stern-Gerlach apparatus, producing a set of peaks. The uppermost peak only is then passed through *another* Stern-Gerlach apparatus (with the same magnetic field gradient dB/dz as the first). How many peaks are observed in the output of the second Stern-Gerlach apparatus?



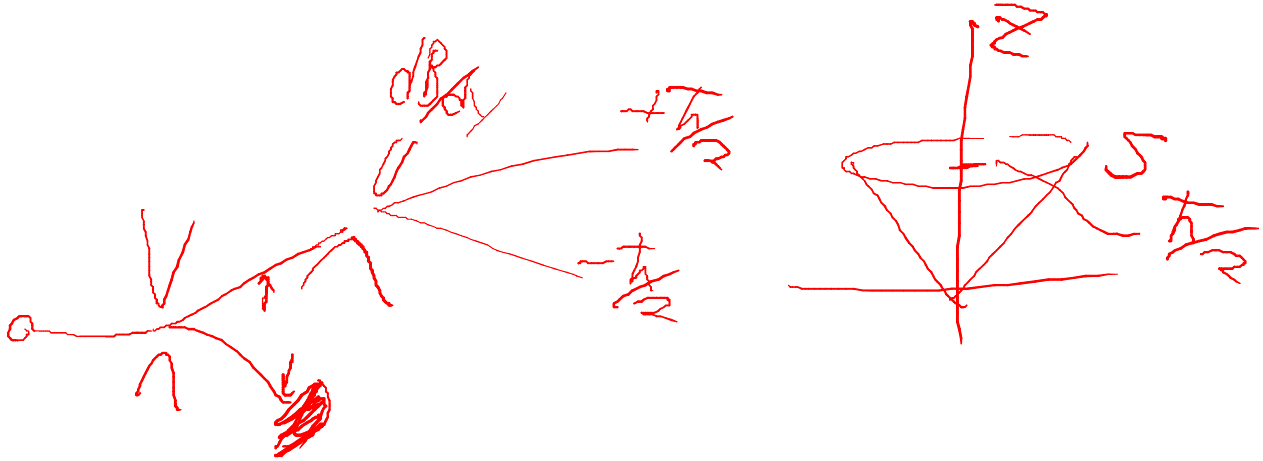
a. 0
b. 1
c. 2
d. 3
e. 4

$$F_z = -\frac{dV}{dz} = -\mu \frac{dB}{dz}$$

$$U = -\vec{\mu} \cdot \vec{B}$$

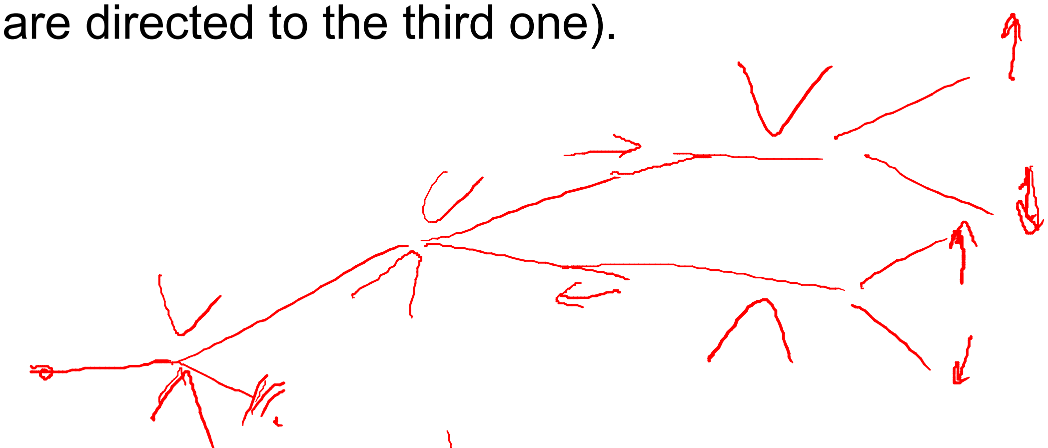
24. If instead we were to rotate the second Stern-Gerlach apparatus by 90°, so that the gradient was dB/dy instead, now how many peaks would be observed?

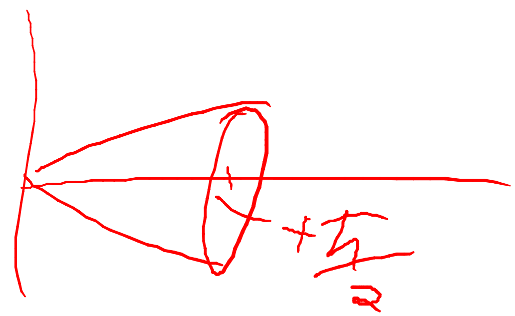
a. 0
b. 1
c. 2
d. 3
e. 4



24’. If after the second Stern-Gerlach apparatus with gradient dB/dy, we now install a third Stern-Gerlach apparatus, again with gradient dB/dz, how many peaks would be observed (assuming all peaks from the 2^{nd} SG apparatus are directed to the third one).

a. 0
b. 1
c. 2
d. 3
e. 4





25. What are the quantum numbers n and l of the outermost electron of a Br atom? Br has 35 electrons.

a. n=3, l=0
b. n=3, l=1
c. n=4, l=0
d. n=4, l=1
e. n=4, l=2

$p \Rightarrow \begin{matrix} \ell = 1 \\ m_\ell = 0, +1, -1 \end{matrix}$

	ℓ
s	0
p	1
d	2
f	3

1s
2s 2p
3s 3p 3d
4s 4p 4d 4f

$1s^2\ 2s^2\ 2p^6$ 10
$3s^2\ 3p^6\ 4s^2$ 10
$3d^{10}$ 10
$4p^5$

26. If the outermost electron is now excited (e.g., by a collision) to the n = 5, l = 1 state, to which final state(s) could the electron fall back down by emitting a photon?

a. n=4, l=3
b. n=4, l=2
c. n=5, l=0
d. n=4, l=1
e. n=3, l=2

orbital already filled

$\Delta \ell = \pm 1$

(because $S_{z,\text{photon}} = \pm \hbar$)

$1s^2$

$2s^2 \quad 2p^6$

$3s^2 \quad 3p^6 \quad 3d^{10}$

$4s^2 \quad 4p^5 \quad 4d \quad 4f$

$5s \quad 5p$

$\ell = 1 \rightarrow \ell = 0$

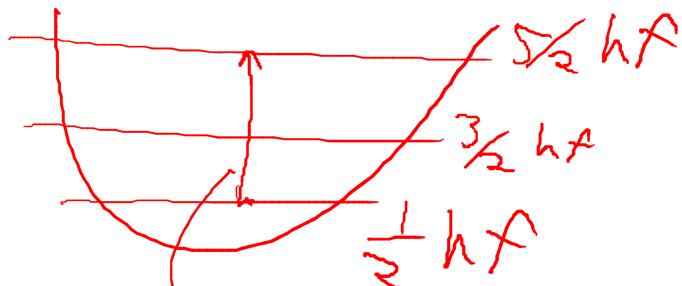
$\ell = 1 \rightarrow \ell = 2$

Problems 27-29 refer to this situation:

A calcium ion (charge |e|, mass = 6.65×10^{-26} kg) is trapped in an electromagnetic potential that approximates a **harmonic oscillator**. The frequency associated with the oscillation of the ion in the trap is 100 kHz. $\times 2$

27. If one wanted to excite the ion from the ground state of the trap *directly* to the *second excited state*, one might shine on radio waves with frequency:

a. 100 kHz
b. 200 kHz
c. 800 kHz



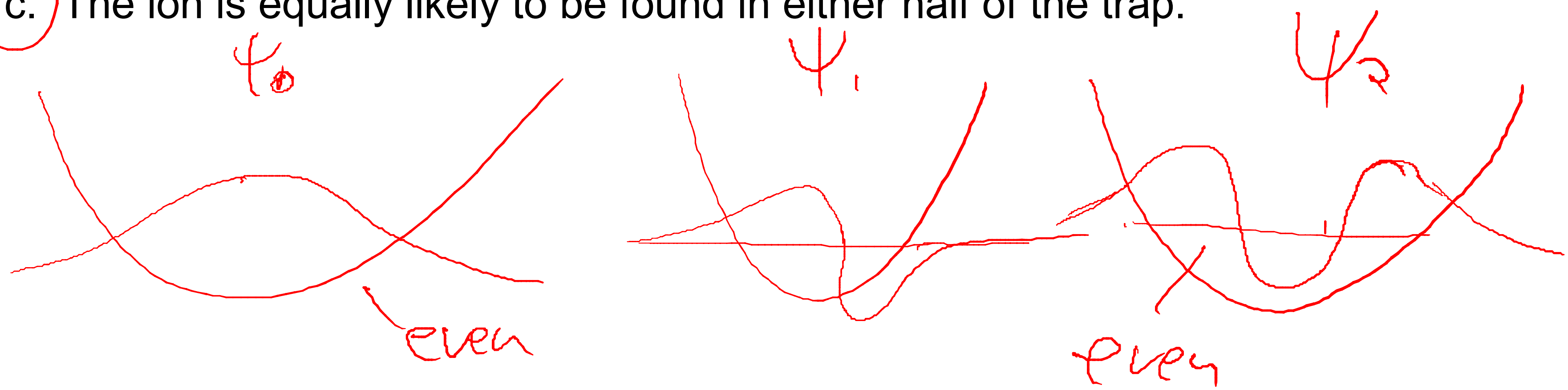
$E_{photon} + E_g = E_{2nd\ excited}$

$E_{photon} = E_{2nd} - E_g = 2hf_{osc} = hf_{photon}$

$f_{photon} = 2f_{osc}$

28. At time t = 0, the ion is prepared into an equal superposition of the ground state and the *second* excited state, $\psi = \frac{1}{\sqrt{2}}(\psi_0 + \psi_2)$. Which of the following describes the likely location of the ion:

a. The ion is more likely to be found in the left-hand side of the trap.
b. The ion is more likely to be found in the right-hand side of the trap.
c. The ion is equally likely to be found in either half of the trap.

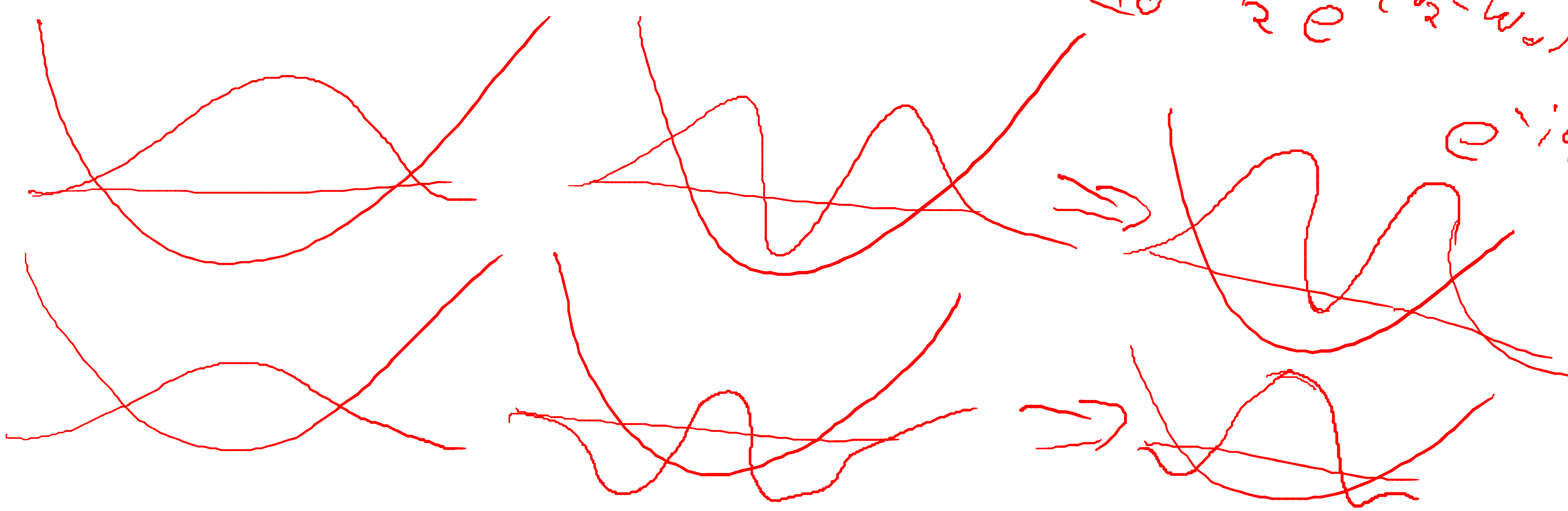


Sym + Sym → Sym

29. We now let the system evolve in time. Which of the following best describes the future behavior of the ion:

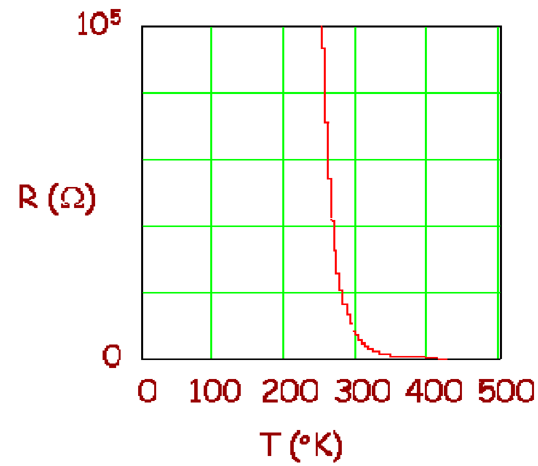
a. The ion will “slosh” back and forth from the left-hand side of the well to the right-hand side.
b. The ion will “slosh” back and forth from being mostly located near the center of the well to being mostly located away from the center (i.e., nearer the “edges” of the well).
c. The probability density of the ion will not change over time.

$$\psi_0 e^{-i\omega_0 t} + \psi_2 e^{-i\omega_2 t} = \left(\psi_0 + \psi_2 e^{-i(\omega_2-\omega_0)t}\right) e^{-i\omega_0 t}$$



30. Consider the following curve of resistance versus temperature.

What kind of material is this?



a. insulator
b. semiconductor
c. Metal

Metal

T ↑

scattering ↑

τ ↓

σ ↓

R ↑ R

T ↑

R ↓

σ ↑

$$\sigma = \frac{n e^2 \tau}{m}$$

Semiconductor